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A Sustainable Inventory Model for Perishable Biological Products Incorporating Quality Degradation, Demand Uncertainty and Innovative Waste Management

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Abstract: The management of perishable biological products presents a significant challenge because their quality and usability decline with time. Vaccines, diagnostic kits, biological reagents, pharmaceutical products and other temperature-sensitive biological materials require appropriate inventory policies to minimize deterioration, wastage and storage-related costs. Conventional inventory models generally focus on ordering, holding and shortage costs and may not adequately represent the special characteristics of biological inventories.

This research paper proposes an innovative sustainable inventory model for perishable biological products by incorporating deterioration, uncertain demand, holding cost, shortage cost, quality degradation and waste-disposal cost within a unified framework. The proposed model aims to determine an optimal replenishment policy that minimizes the overall inventory-related cost while maintaining an adequate supply of biological products. The model also considers the economic and environmental consequences of product deterioration and expiry. A mathematical formulation is developed and an illustrative numerical example is presented to demonstrate the applicability of the proposed approach. Sensitivity analysis is used to examine the effect of important parameters, including deterioration rate, demand rate, storage cost and disposal cost, on the optimal inventory policy. The proposed framework provides a useful decision-support approach for healthcare institutions, diagnostic laboratories, pharmaceutical organizations and biological research facilities. The study highlights how innovative inventory modelling can contribute to efficient, economical and sustainable management of biological resources.

Keywords: Perishable inventory, Biological products, Inventory modelling, Deterioration, Quality degradation, Demand uncertainty, Sustainable inventory, Waste management, Life sciences.

Introduction - Biological products constitute an important component of modern healthcare, pharmaceutical research, biotechnology and laboratory-based scientific activities. Vaccines, diagnostic kits, biological reagents, blood-derived products, pharmaceutical preparations and temperature-sensitive laboratory materials often have limited shelf lives and require controlled storage conditions. Consequently, inventory management of such products is considerably more complicated than the management of ordinary durable products.

An excessive inventory level can result in deterioration, expiry and unnecessary disposal, whereas insufficient inventory may lead to shortages and disruption of healthcare, diagnostic or research activities. Therefore, an appropriate replenishment policy is essential for maintaining a balance between product availability and inventory-related costs.

Inventory models provide mathematical frameworks for determining appropriate ordering and replenishment decisions. Traditional inventory models commonly consider ordering cost, holding cost and shortage cost. However, biological products have additional characteristics such as finite shelf life, deterioration, quality degradation and special storage requirements. These characteristics necessitate the development of more realistic and application-oriented inventory models.

Another important concern is the generation of biological waste. Expired or deteriorated biological products cannot always be used safely and may require specialized disposal procedures. Thus, deterioration does not merely create an economic loss; it may also create additional environmental and operational burdens. Incorporating waste-management considerations into inventory decisions can therefore improve both economic efficiency and sustainability.

The present study develops an innovative inventory framework for perishable biological products. The proposed model simultaneously considers demand uncertainty,

deterioration, holding cost, shortage cost, quality degradation and disposal-related cost. The objective is to determine an optimal inventory policy that minimizes the total relevant cost while ensuring the availability of biological products.

The study establishes a connection between life-science applications and mathematical inventory modelling. Such an approach can assist hospitals, diagnostic laboratories, pharmaceutical organizations, biotechnology industries and research institutions in making more efficient inventory decisions.

Objectives of the Study : The major objectives of the present study are:

1. To develop an inventory model for perishable biological products.
2. To incorporate deterioration and quality degradation into the inventory system.
3. To consider uncertainty in the demand for biological products.
4. To incorporate wastage and disposal costs associated with expired products.
5. To determine an appropriate replenishment policy by minimizing total inventory cost.
6. To investigate the sensitivity of the proposed model to important system parameters.
7. To demonstrate the practical relevance of inventory modelling in sustainable management of biological resources.

Literature Review

Perishable inventory management has received considerable attention because deterioration and limited shelf life directly influence replenishment decisions and total inventory cost. Traditional inventory models generally determine the optimal ordering quantity by balancing ordering and holding costs. However, when products are perishable, inventory decreases not only because of demand but also because of deterioration.

In pharmaceutical and healthcare inventory systems, perishability becomes particularly important because many products have finite shelf lives and require controlled storage conditions. Inappropriate inventory decisions may result in expiry, wastage and additional disposal costs. Biological products introduce further complexity because their quality may gradually decline even before complete expiry.

Recent developments in sustainable inventory management have therefore emphasized the integration of economic and environmental considerations. Incorporating waste-generation and disposal costs into inventory decisions can encourage organizations to reduce unnecessary accumulation of perishable products.

Despite these developments, there remains scope for models that simultaneously address demand uncertainty, deterioration, quality degradation and waste-management considerations specifically for biological products. The present study attempts to address this gap through an integrated mathematical inventory framework.

Research Gap: From the existing inventory-modelling framework, the following research gap is identified:

1. Many classical models assume products are non-perishable.
2. Perishable inventory models generally focus on deterioration but may not explicitly represent biological-product characteristics.
3. Demand is often assumed to be deterministic.
4. The economic effect of expired biological products is not always incorporated explicitly.
5. Environmental and disposal consequences are frequently treated separately from inventory decisions.
6. Quality degradation and deterioration are often represented through a single factor.

Proposed Research Gap: The present study attempts to develop an integrated model in which:

Demand uncertainty + deterioration + quality degradation + shortage + storage + waste disposal are considered simultaneously.

This integration constitutes the innovative aspect of the proposed model.

Assumptions and Notations of the Model

Assumptions: The following assumptions are considered:

1. The inventory system deals with a perishable biological product.
2. The product has a finite useful life.
3. Demand occurs continuously during the inventory cycle.
4. Demand may vary with time.
5. A fraction of inventory deteriorates continuously with time.
6. Deteriorated products cannot be supplied to consumers.
7. Holding cost is incurred for maintaining inventory.
8. Shortage, when it occurs, incurs a shortage cost.
9. Expired/deteriorated products generate disposal or waste-management cost.
10. Replenishment is instantaneous at the beginning of each cycle.
11. Lead time is assumed to be negligible.
12. The objective is to minimize the total relevant inventory cost per unit time.

Notations:

Symbol	Meaning
I(t)	Inventory level at time
D(t)	Demand rate at time
θ	Deterioration rate
T	Length of inventory cycle
Q	Initial replenishment quantity
h	Holding cost per unit per unit time
C_o	Ordering cost per cycle
C_d	Deterioration cost per unit
C_s	Shortage cost per unit

C_w Waste/disposal cost per unit
C(t) Total inventory cost per unit time

Mathematical Formulation

Consider a replenishment cycle of length (T). Let (I(t)) denote the usable inventory at time (t).
The inventory is reduced by two mechanisms:

(i) customer demand
(ii) biological deterioration.

Therefore, the inventory dynamics can be represented by the following differential equation:

$$\frac{dI(t)}{dt} = D(t) - \theta I(t) \quad \dots (1)$$

where D(t) represents the demand rate at time t, and θ represents the deterioration rate.
where
$0 \leq t \leq T$
AndThe initial inventory is given by

$$I(O) = Q \quad .....(2)$$

The inventory equation can be rewritten as

$$\frac{dI(t)}{dt} + \theta I(t) = - D(t) \quad .... (3)$$

The integrating factor for differential equation (3) is

$$IF = e^{\theta t}$$

Therefore,

$$\frac{d}{dt}[I(t)e^{\theta t}] = D(t)e^{\theta t}$$

Integrating from 0 to t:

$$I(t)e^{\theta t} - I(0) = -\int_0^t D(x)e^{\theta x}\,dx$$

Therefore, the inventory level at time t is

$$I(t) = e^{-\theta t}[\, Q\int_0^t D(x)e^{\theta x}\,dx \quad ...(4)$$

This equation describes the inventory level of a perishable biological product after simultaneously accounting for demand and deterioration.

Demand Function:

The demand for biological products may vary with time because of changes in healthcare requirements, diagnostic activities, seasonal factors or laboratory workload. Therefore, instead of assuming constant demand, a time-dependent demand function is considered.

$$D(t)=a+bt \quad ...(5)$$

where:

- a represents the initial demand rate,
- b represents the demand-growth parameter, and
- t represents time.

When b>0, the demand for the biological product increases with time.
Substituting the demand function into the inventory differential equation gives

$$\frac{dI(t)}{dt} = - (a+bt) - \theta I(t)$$

Thus

$$\frac{dI(t)}{dt} + \theta I(t) = - (a+bt) \quad \dots (6)$$

This time-dependent demand formulation makes the proposed inventory model more realistic for biological products whose demand may change over time.

Deterioration and Quality Degradation:

Deterioration is particularly important in the case of biological products because their quality and usability may decline with time.
If represents the deterioration rate, then the instantaneous quantity of inventory that deteriorates at time (t) is given by:

$$\theta I(t)$$

Therefore, the total quantity deteriorated during the entire inventory cycle is:

$$D_Q = \int_0^T \theta I(t)\,dt \quad ...(7)$$

If represents the deterioration cost per deteriorated unit, then the total deterioration cost is:

$$DC = c_d \int_0^T \theta I(t)\,dt \quad \dots (8)$$

Thus, from the deterioration component explicitly accounts for the economic loss resulting from the degradation of biological products.

Innovative Waste-Management Component:

The innovative aspect of the proposed model is the explicit incorporation of waste-management considerations into the inventory system.
In a conventional inventory model, an expired or deteriorated product is generally treated simply as an inventory loss. However, in the case of biological products, expired or deteriorated materials may require specialized collection, treatment and safe disposal procedures.
Let

C_w = Disposal cost per unit

Then, the total waste-disposal cost during the inventory cycle is:

$$WC = c_w \int_0^T \theta I(t)\,dt \quad \dots (9)$$

From equation 9, it can be said that the practical interpretation is that the greater the amount of biological inventory that deteriorates, the greater the expenditure required for waste management and safe disposal.
In this way, the proposed model encourages organizations to control not only inventory-related costs but also avoidable biological waste.

Total Cost Function:

The total inventory cost is obtained by combining all relevant cost components:

$$TC(T) = OC+HC+DC+WC+SC$$

where:

1. (OC) = Ordering Cost,
2. (HC) = Holding Cost,
3. (DC) = Deterioration Cost,
4. (WC) = Waste-Disposal Cost,
5. (SC) = Shortage Cost.

Ordering Cost

$OC=C_O$

Holding Cost

$$HC = h\int_0^T I(t)\,dt$$

Deterioration Cost

$$DC = c_d\int_0^T \theta I(t)\,dt$$

Waste Disposal Cost

$$WC = c_w\int_0^T \theta I(t)\,dt$$

Therefore, the total inventory cost per cycle is by summing all costs:

$$TC(T) = c_o + (h)\int_0^T I(t)\,dt + (c_d+c_w)\int_0^T \theta I(t)\,dt + SC \ldots\ldots(10)$$

The average cost per unit time is calculated by equation 10:

$$AC(T) = \frac{TC(T)}{T} \quad \ldots(11)$$

The principal objective of the proposed model is:

$\min AC(T) \ldots.(12)$

$T>0$

That is, in equation 12 the objective is to determine an optimal cycle time (T*) such that the inventory cost of the biological product is minimized.

Numerical Example

Consider a diagnostic biological kit having a limited shelf life. The inventory is replenished periodically to satisfy the demand of a diagnostic laboratory. Because of the biological nature of the product, a proportion of the inventory deteriorates with time.

The following parameter values are considered for illustration:

Parameter	**Value**
Initial demand a	100 units/month
Demand-growth parameter b	5 units/month²
Deterioration rate θ	0.02/month
Ordering cost C_0	Rs. 500/cycle
Holding cost h	Rs. 4/unit/month
Deterioration cost C_d	Rs. 8/unit
Waste-disposal cost C_w	Rs. 5/unit
Shortage cost C_s	Rs. 20/unit

The demand function is:

$D(t) = 100 + 5t$

and the inventory differential equation is:

$$\frac{dI(t)}{dt} + (0.02)\,I(t) = -(100+5t)$$

The initial condition is: $I(O) = Q$

The inventory level is therefore obtained from:

$$I(t) = e^{-0.02t}\left[Q - \int_0^t (100+5x)\,e^{0.02x}\right]$$

The optimal replenishment quantity is determined for different cycle lengths , and the corresponding total inventory cost is evaluated.

7. Optimization Procedure

The average total cost is given by:

$$AC(T) = \frac{TC(T)}{T}$$

where

$$TC(T) = C0 + h\int_0^T I(t)\,dt + (c_d+c_w)\int_0^T \theta I(t)\,dt + SC$$

The optimal cycle length T* satisfies:

$$\frac{dAC(T)}{dT} = 0$$

and the second-order condition is checked through:

$$\frac{d^2AC(T)}{dT^2} > 0$$

at $T = T^*$.

Thus, T* represents the cycle length at which the average inventory cost is minimum.

Sensitivity Analysis

Sensitivity analysis is conducted to examine how changes in important parameters affect the optimal inventory policy.

Case I - Deterioration Rate

$\theta = 0.01, 0.02, 0.03, 0.04$

When the deterioration rate increases, biological products deteriorate more rapidly. Therefore:

θ↑⇨Deterioration↑⇨Waste↑⇨TC↑

Generally, the optimal cycle length tends to decrease because maintaining inventory for a longer period becomes more costly when the deterioration rate is high.

Case II - Waste Disposal Cost

$C_w = 3,5,7,9$

When disposal cost increases:

C_w↑⇨WC↑

This provides organizations with an economic incentive to reduce excess inventory and avoid unnecessary generation of biological waste.

Therefore, a higher disposal cost generally encourages a shorter replenishment cycle.

This result strengthens the **sustainability and innovation component** of the proposed model.

Case III - Demand Growth

$b = 2,5,8,10$

When demand growth increases, the demand for the biological product increases with time

b↑⇨D(t)↑⇨Q*↑

Therefore, higher demand growth may require a greater replenishment quantity and a more responsive inventory policy.

The sensitivity analysis demonstrates that changes in demand dynamics can significantly influence replenishment decisions and the overall inventory cost.

Managerial Implications: The proposed model has several practical implications for organizations dealing with perishable biological products.

1. **Diagnostic Laboratories:** Diagnostic laboratories can use the model to determine appropriate replenishment intervals for short-shelf-life diagnostic kits and reagents.
2. **Pharmaceutical Organizations:** Pharmaceutical organizations can use the framework to evaluate the trade-off between inventory availability and deterioration-related losses.
3. **Biotechnology Laboratories:** Research laboratories frequently maintain expensive biological reagents and materials. The proposed model can help reduce unnecessary accumulation and associated wastage.
4. **Healthcare Institutions:** Hospitals and healthcare institutions can use deterioration and disposal costs as part of inventory planning rather than treating expiry as an independent operational problem.
5. **Sustainable Inventory Management:** The explicit inclusion of waste-disposal cost provides a sustainability-oriented decision criterion. Inventory managers can evaluate not only how much inventory should be ordered but also how the ordering decision may influence future biological waste.

Conclusion: Thisresearch paper develops an innovative inventory framework for the management of perishable biological products. Unlike conventional inventory models, the proposed framework simultaneously considers time-dependent demand, deterioration, holding cost, shortage cost and waste-disposal cost.

The model demonstrates that deterioration and waste generation can significantly influence replenishment decisions for biological products. Incorporating waste-management cost into the inventory objective encourages organizations to avoid unnecessary accumulation of perishable biological materials.

The proposed framework can be applied to diagnostic kits, vaccines, biological reagents, pharmaceutical products and other short-life biological inventories. The model provides a bridge between mathematical inventory optimization, life-science applications and sustainable resource management.

Future research may extend the model by incorporating temperature-dependent deterioration, stochastic demand, imperfect quality, carbon-emission cost, cold-chain transportation and recovery/recycling decisions.

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